

Mode shape description of an aero-engine casing structure using Zernike moment descriptors

LIU Ying-chao, ZANG Chao-ping

(College of Energy and Power Engineering,
Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China)

Abstract: Vibration mode shape description of an aero-engine casing structure using Zernike moment descriptor (ZMD) was introduced in this paper. The mode shapes of the aero-engine casing structure can be decomposed as a linear combination of a series of Zernike polynomials, with the feature of each Zernike polynomial reflecting a part of characteristic of mode shapes, based on Zernike moment transformation. Meanwhile, the reconstruction of mode shapes with ZMD was explored and its ability to filtering the noise contaminated in the mode shapes was studied. Simulation of the aero-engine casing structure indicated the advantage of this method to depict the mode shapes of a symmetric structure. Results demonstrate that the Zernike moment description of the mode shapes can effectively describe the double modes in the symmetric structure and also has the ability to remove or significantly reduce the influence of noise in the mode shapes. Such feature shows great practical value for further research on the correlation, model updating and model validation of the symmetric structure's finite element model.

Key words: Zernike; mode shape; modal assurance criterion (MAC); reconstruction; noise filtering

CLC number: O174.43; V231.92

Document code: A

Introduction

In structural dynamics, correlation analysis between prediction from finite element analysis and measured data is a crucial step in model validation and updating^[1-3]. The current method, the modal assurance criterion (MAC)^[4], is an indicator of the closeness of a finite element prediction to a measured mode and can be interpreted as the cosine of the angle between the numerical eigenvector and a measured mode shape. In recent years, laser-based techniques such as continuous scanning laser Doppler vibrometry (CSLDV) and electronic speckle pattern interferometry (ESPI) have been applied in structural dynamics^[5-6]. With this new technology, a

large number of degrees of freedom (DOFs) can be measured and more accurate mode shapes can be achieved in a relatively short testing time.

In the research of European Commission Framework 6 Integrated Project VIVACE (Value Improvement through a Virtual Aeronautical Collaborative Enterprise), Zang et al. made a comprehensive assessment of using full-field vibration measurement in structural dynamics, and suggested further requirements for development of such technique^[7]. A grounded plate used as an example to demonstrate the benefits of using detailed mode shapes for model correlation and updating in the validation process^[8], and the finite element (FE) model updating is achieved using measured data obtained from

Received: 2010-11-15; **revision received:** 2011-03-07

Foundation item: Supported by Research Fund for the Doctoral Program of Higher Education of China (20093218110008);

The SRF for ROCS, SPM (No. R0861-21);

Jiangsu Research Foundation of Talented Scholars in Six Fields (No. P0951-021);

The Nanjing University of Aeronautics and Astronautics Postgraduate Innovation Fund

E-mail: c. zang@nuaa.edu.cn

CSLDV technique^[9]. Schwingshackl et al. measured the dynamic behavior of a cylindrical structure (an aero-engine casing) using CSLDV technique with a previously unachievable spatial resolution in a much shorter time than conventional measurement methods^[10-11]. However, it is still under research how to effectively use these detail and redundant modeshape data in the model validation process.

Research found that the MAC index encapsulates all the information contained in the difference between measured and predicted modes, which means that an appreciation of subtle changes in shapes, either locally or globally, is clearly unobtainable from the MAC value in the case of large and complicated industrial systems. In addition, for a symmetric structure, MAC also has the difficulty to distinguish the repeated modes which have same frequencies but different mode shapes.

Zernike moment is based on a set of complex polynomials that form a complete orthogonal set over the interior of the unit circle. It is found that the Zernike moment is one of the most important region-based shape descriptors because of its outstanding properties, including (1) rotational invariance^[12-13], (2) robustness to noise^[14], (3) reconstruction is easy and (4) computing efficiency. Therefore, it is widely used in image processing and pattern recognition^[15]. Most recently, the method of mode shape recognition and finite element model updating using the Zernike moment descriptors (ZMDs) is reported in the Ref. [16] for the first time and a two-dimensional (2-D) disk with one direction vibration is used as an example. As an extension, we will introduce an approach to deal with a three-dimensional (3-D) symmetric structure — an aero-engine casing, with vibrations in the 3 directions. The resultant mode shapes of the structure are first divided into the x, y, z directions and the mode shapes in each direction are decomposed as a linear combination of a series of Zernike polynomials, with the feature of each Zernike polynomial reflecting a part of character-

istic of mode shapes, based on Zernike moment transformation. The reconstruction of resultant mode shapes are integrated from those in the x, y, z directions. Simulation results demonstrate the superiority of describing symmetrical structural mode shapes using Zernike moment descriptor (ZMD). The ability of filtering the noise contaminated in the mode shapes of proposed method is also discussed.

This paper is organized as follows. In section 1, methods of describing mode shape using ZMDs and reconstruction of mode shape are given. Section 2, the case study, an example of simplified aero-engine casing structure demonstrates the superiority of ZMDs and effectiveness of filtering the noise contaminated in mode shapes. Finally, the summary of total paper is given.

1 Mode shapes description using ZMD

1.1 Zernike moment descriptor

Generally, Zernike moments are based on a set of complex polynomials that form a complete orthogonal set over the interior of the unit circle. They are defined as the projection of the image on these orthogonal basis functions. Specifically, the basis functions $V_{n,m}(\rho, \theta)$ are given by

$$V_{n,m}(\rho, \theta) = R_{n,m}(\rho) e^{im\theta} \quad (1)$$

Where $i = \sqrt{-1}$, n is a non-negative integer, representing the order of the radial polynomial, m is positive or negative integers subjects to constraints $n - |m|$ even, and $n \geq |m|$, representing the repetition of the azimuthal angle. ρ is the length of vector from the origin to (x, y) . θ is the azimuthal angle between vector ρ and the x -axis in counterclockwise direction.

Radial polynomial $R_{n,m}$ is defined in Eq. (2):

$$R_{n,m}(\rho) = \sum_{s=0}^{\frac{n-|m|}{2}} \frac{(-1)^s (n-s)! \rho^{n-2s}}{s! \left(\frac{n+|m|}{2} - s \right)! \left(\frac{n-|m|}{2} - s \right)!} \quad (2)$$

As the complex Zernike polynomials are orthogonal, they will satisfy the orthogonality condition as below:

$$\iint_{x^2+y^2 \leq 1} V_{p,q}(\rho, \theta) V_{n,m}^*(\rho, \theta) dx dy = \frac{\pi}{n+1} \delta_{n,p} \delta_{m,q} \quad (3)$$

where ‘*’ denotes the complex conjugate and $\delta_{n,p}$, $\delta_{m,q}$ are Kronecker deltas that satisfy:

$$\delta_{i,j} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (4)$$

Hence, the Zernike moment of order n with repetition m for a mode shape $\psi(x, y)$ of a structure is given by

$$Z_{n,m} = \frac{n+1}{\pi} \iint_{x^2+y^2 \leq 1} \psi(x, y) V_{n,m}^*(x, y) dx dy \quad (5)$$

The formula can also be expressed as following in the polar coordinates,

$$Z_{n,m} = \frac{n+1}{\pi} \int_0^{2\pi} \int_0^1 \psi(\rho, \theta) V_{n,m}^*(\rho, \theta) \rho d\rho d\theta \quad (6)$$

The ZMD of a mode shape of the structure may be calculated by replacing the integral in Eq. (5) by the summation.

$$Z_{n,m} = \sum_x \sum_y \psi(x, y) V_{n,m}^*(x, y) \quad (7)$$

1.2 Rotation invariance of ZMD

ZMD has the feature of rotation invariance. Let the mode shape $\psi'(\rho, \theta)$ be the rotated mode shape of $\psi(\rho, \theta)$ by angle α about its origin, that is

$$\psi'(\rho, \theta) = \psi(\rho, \theta - \alpha) \quad (8)$$

The Zernike moment of the rotated mode shape in the polar coordinate is,

$$Z'_{n,m} = \frac{n+1}{\pi} \sum_{\rho} \sum_{\theta} V^*(\rho, \theta) \psi(\rho, \theta - \alpha) \quad (9)$$

or

$$Z'_{n,m} = Z_{n,m} e^{-m\alpha} \quad (10)$$

Here, the magnitude of the Zernike moment of the rotated mode shape and that of the reference are the same, i. e.,

$$|Z'_{n,m}| = |Z_{n,m}| \quad (11)$$

Whenever the Zernike moment is known, the rotational angle can be determined as,

$$\alpha = \frac{\arg(Z'_{n,m}/Z_{n,m})}{m} \quad (12)$$

where m is the repetition of the azimuthal angle.

1.3 Reconstruction of mode shapes

According to Fourier expansion theorem of

the orthogonal function, the mode shape $\psi(\rho, \theta)$ of the structure can be reconstructed by limited orders of ZMDs in Eq. (13).

$$\psi(\rho, \theta) \cong \sum_{p=0}^{N_{\max}} \left\{ \frac{Z_{p,0}^{(\text{Re})}}{2} R_{p,0}(\rho, \theta) + \sum_{q>0} \left[Z_{p,q}^{(\text{Re})} \cos(q\theta) + Z_{p,q}^{(\text{Im})} \sin(q\theta) R_{p,q}(\rho, \theta) \right] \right\} \quad (13)$$

where $Z_{p,q}^{(\text{Re})}$ and $Z_{p,q}^{(\text{Im})}$, respectively, is the real and imaginary of the Zernike moment, and may be determined as,

$$Z_{n,m}^{(\text{Re})} = \frac{2(n+1)}{\pi} \int_0^{2\pi} \int_0^1 \psi(\rho, \theta) R_{n,m}(\rho, \theta) \cos(m\theta) \rho d\rho d\theta; \quad (m \geq 0) \quad (14)$$

$$Z_{n,m}^{(\text{Im})} = \frac{2(n+1)}{\pi} \int_0^{2\pi} \int_0^1 \psi(\rho, \theta) R_{n,m}(\rho, \theta) \sin(m\theta) \rho d\rho d\theta; \quad (m \geq 0) \quad (15)$$

The relationship between $Z_{p,q}^{(\text{Re})}$ and $Z_{p,q}^{(\text{Im})}$ with $Z_{n,m}$ is defined as,

$$Z_{p,q}^{(\text{Re})} = 2 \text{Re}(Z_{n,m}) \quad (16)$$

$$Z_{p,q}^{(\text{Im})} = -2 \text{Im}(Z_{n,m}) \quad (17)$$

where, $\text{Re}(Z_{n,m})$ and $\text{Im}(Z_{n,m})$ is real and imaginary of $Z_{n,m}$ respectively.

A correlation coefficient based on mean-square error can be defined as,

$$\text{corr}(\psi', \psi) = \frac{\iint_{\Omega} (\psi' - \bar{\psi}')(\psi - \bar{\psi}) dA}{\sqrt{\left[\iint_{\Omega} (\psi' - \bar{\psi}')^2 dA \right] \left[\iint_{\Omega} (\psi - \bar{\psi})^2 dA \right]}} \quad (18)$$

where, ψ' and ψ denote reconstructed mode shape and original one respectively.

And,

$$\bar{\psi}' = \frac{\int_{\Omega} \psi' dA}{\int_{\Omega} dA} \quad (19)$$

$$\bar{\psi} = \frac{\int_{\Omega} \psi dA}{\int_{\Omega} dA} \quad (20)$$

The value of $\text{corr}(\psi', \psi)$ in Eq. (18) is a real one between 0 and 1. A value of 1 means that reconstructed and original mode shapes are exactly the same.

1.4 Determination of ZMD shape feature set

Based on Zernike moment transformation, mode shape data can be decomposed into a linear combination of Zernike polynomials which constitute the Zernike moment shape feature set: $\mathbf{Z} = \{Z_{0,0}, Z_{1,1}, Z_{2,0}, Z_{2,2}, Z_{3,1}, Z_{3,3}, Z_{4,0}, Z_{4,2}, Z_{4,4}, \dots\}$. Each entry reflects a part of characteristic of mode shapes. Approximate reconstruction may be achieved with proper choice of shape feature set. Similarly, because of the property of robustness to noise, the systematic noise and measurement error may be eliminated or decreased if proper shape feature set is chosen.

One of the major concerns is how to determine proper shape feature set, in another word, how we choose the maximum order of ZMDs, $N_{\max}$, when the mode shape is decomposed into a linear combination of Zernike polynomials. A reasonable method may be as follow: set an initial value which may be 3 as the maximum order, then compute ZMDs of mode shapes and shape feature set will be determined. Next, calculate the correlation between original and reconstructed mode shapes; if the value of correlation is larger than a given threshold δ , we consider that the maximum order is which we wanted. If not, the initial value increases 1, previous processing is redone again till the value meets requirement. The flowchart is shown in Fig. 1.

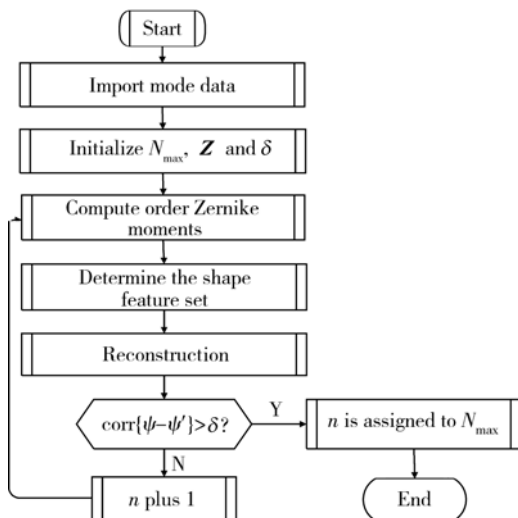


Fig. 1 Flowchart of determining maximum order of ZMDs

2 Case study: an aero-engine casing structure

In order to testify the capability and advantage of dealing with the double modes of symmetric structures using ZMDs, a symmetrically cylindrical structure, an aero-engine casing structure, is used as an example. The details are described as below.

2.1 Finite element model of the aero-engine casing structure

The finite element model of an aero-engine casing structure is displayed in Fig. 2. It is created with four-node quadrilateral plate elements with 1320 nodes. The density and modulus of elasticity of material properties are 7800 kg/m^3 and $2.0 \times 10^5 \text{ MPa}$, respectively. The boundary condition is defined as free-free. The first 16 modes of the model are calculated using the ANSYS solver. The natural frequencies are listed in Table 1 and the corresponding mode shapes are plotted in Fig. 3.

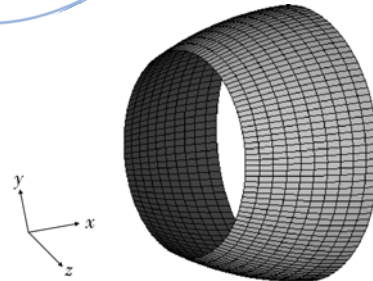


Fig. 2 Finite element model of a simplified aero-engine casing structure

Table 1 Nature frequencies of first 16 normal modes

No. of mode	Natural frequency	No. of mode	Natural frequency
1	30.87	9	201.89
2	30.87	10	201.89
3	80.51	11	238.20
4	80.51	12	238.20
5	82.93	13	335.83
6	82.93	14	335.83
7	152.92	15	353.92
8	152.92	16	353.92

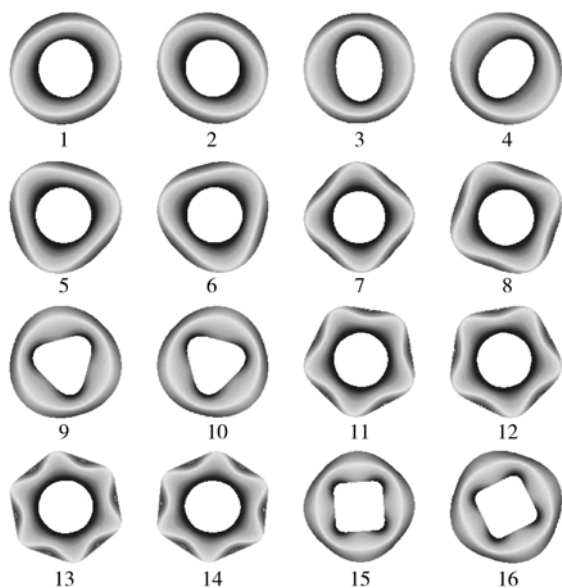


Fig. 3 Mode shape patterns of the aero-engine casing structure in the free-free condition

Clearly, modes 1 and 2 are double modes with same natural frequencies but rotational difference with respect to the symmetric axis of structure (x -axis). Modes 3 and 4, 5 and 6, etc are the same.

In order to apply Zernike moment to the analysis of 3-D structure, the nodes are projected to $y-z$ plane, and treat the mode shape data of different direction of each node as the gray scale of image. The mode shape patterns of three directions are obtained using present method, as Fig. 4 to Fig. 6 shown.

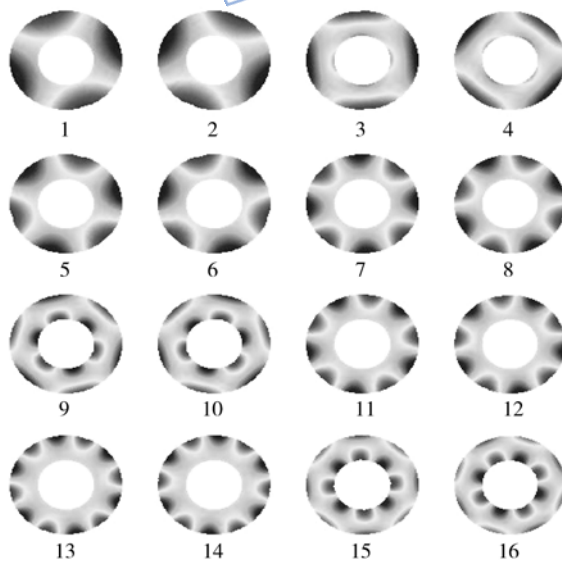


Fig. 4 Mode shape patterns of the aero-engine casing in x direction

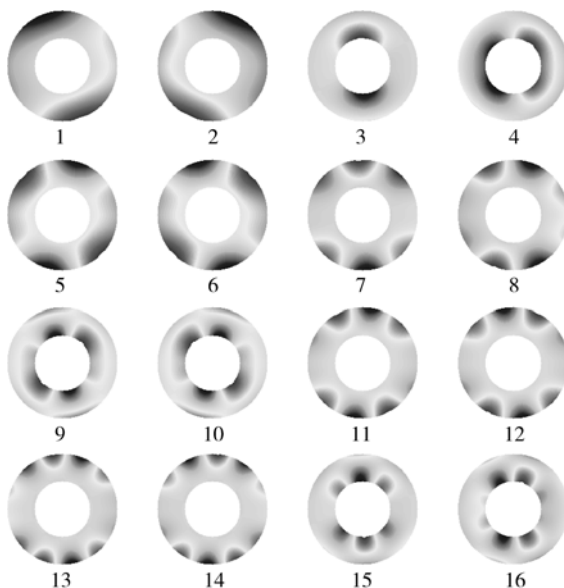


Fig. 5 Mode shape patterns of the aero-engine casing in y direction

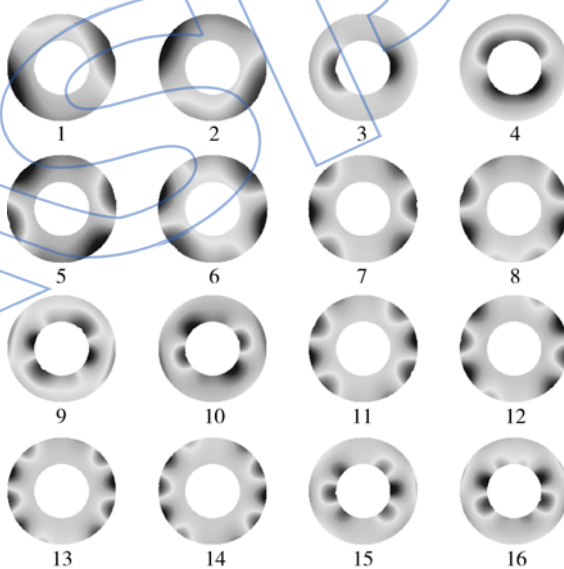


Fig. 6 Mode shape patterns of the aero-engine casing in z direction

In each direction, the double modes of the casing structure using ZMDs are analyzed. Here is the illustration of modeshape patterns in the x direction.

The AUTOMAC of the first 16 normal modes of the simplified aero-engine casing is plotted in Fig. 7. Values of the diagonal matrix are close to 1 and most of those excluding the diagonals are close to 0. There are some relatively values close to 0.4, for instance, the AUTOMAC values between mode 1 and mode 3. It indicates a degree of correlation of both mode

shapes. The reason may be that the modes are supposed to be ‘orthogonal’ and the orthogonality property is only strictly applicable when the mass matrix is used. AUTOMAC indicates that the number of DOFs selected from the FE model is sufficient to discriminate between different mode shapes. However, there is no indication of double modes from the AUTOMAC plot.

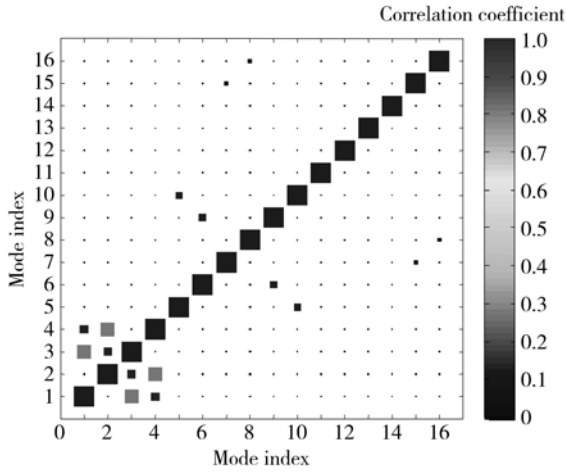


Fig. 7 AUTOMAC of the simplified aero-engine casing

2.2 Mode shape description using ZMD

In order to express the original mode shapes using ZMD, The Zernike transformation of the mode shapes is carried out. Initially the number of the maximum order of ZMDs is chosen as 3 and the threshold of the match between the original mode shapes and the reconstructed ones is set to 0.99. Following the flowchart of determining maximum order of ZMDs, the number of the maximum order of ZMDs is selected to 8 ($N_{\max} = 8$) for the description of the first 16 modes and their correlations between reconstructed and original mode shapes is plotted in Fig. 8. The shape feature set is displayed in Fig. 9. The iterative process costs nearly 10 minutes.

Not only the correlation of mode shapes, but also rotational difference between double modes can be obtained using ZMDs. Fig. 10 shows the amplitudes of Zernike moments of the first 16 modes of the aero-engine casing structure. As can be seen that only several orders of each mode have large values reflecting significant

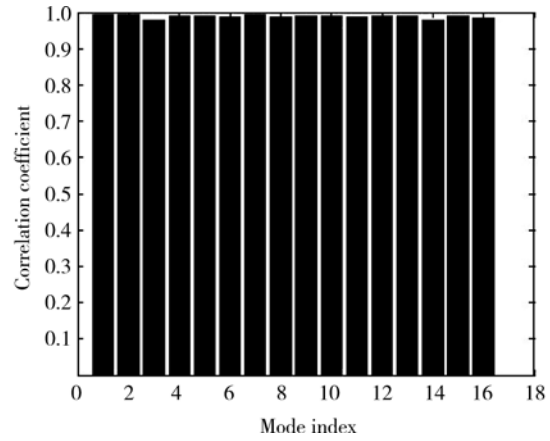


Fig. 8 Correlations between reconstructed and original mode shapes

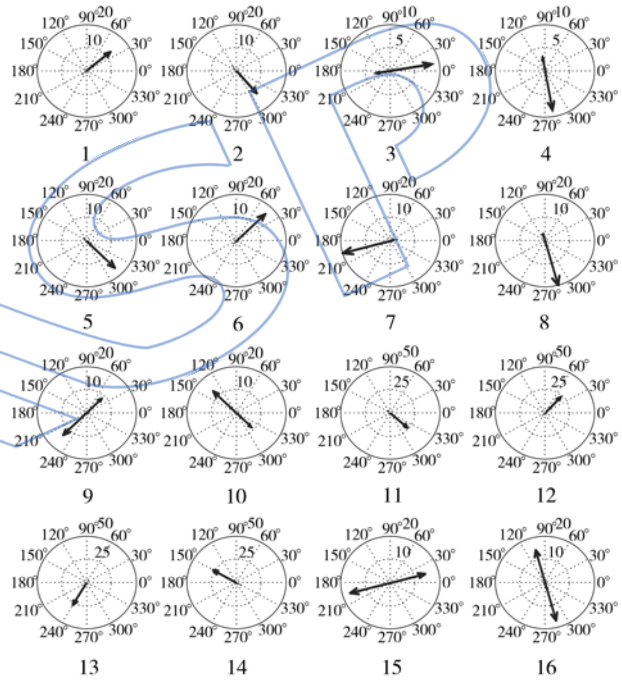


Fig. 9 Shape feature set of the aero-engine casing

characteristic of the shape feature set, the other orders are particular small, so not shown in the figure. According to Eq. (11), the magnitude of Zernike moments extracted from double modes remains unchanged, regardless of their rotational difference. Analogous to MAC, the correlation coefficient of mode shape vectors is defined by the Zernike moments as:

$$Z_{\text{mac}}(r, q) = \frac{|\mathbf{Z}_r \mathbf{Z}_q^T|^2}{(\mathbf{Z}_r \mathbf{Z}_r^T)(\mathbf{Z}_q \mathbf{Z}_q^T)} \quad (21)$$

where r and q denote the mode number, $\mathbf{Z}_r$ and $\mathbf{Z}_q$ denote the Zernike moment magnitude of the r^{th} and q^{th} mode shape respectively, supposed as

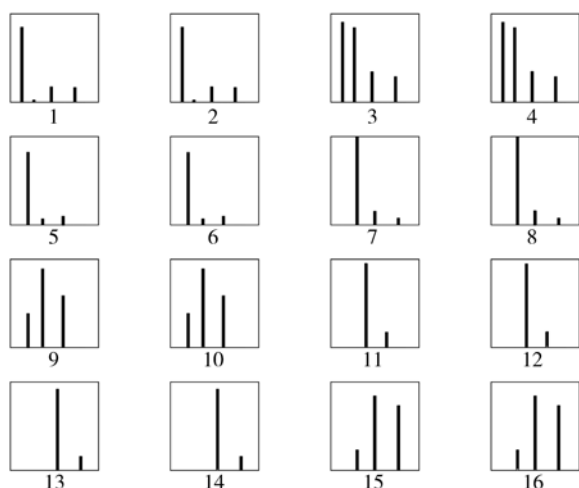


Fig. 10 Amplitudes of ZMDs of the simplified aero-engine casing ($N_{\max}=8$)

row vector. T denotes the transpose.

The autocorrelation coefficients of mode shapes are calculated, as Fig. 11 shown. It is clearly that the autocorrelation coefficient of double mode shapes is closed to 1, i. e. their shapes are 100% correlated, and the double mode shape has almost zero correlation with any other mode shapes. This result is really different with AUTOMAC, the double mode shapes of aero-engine casing are reveal by the amplitudes of the ZMDs. A trivial spatial aliasing is also shown. Obviously, there is particular superiority using ZMDs describing mode shapes of symmetric structure.

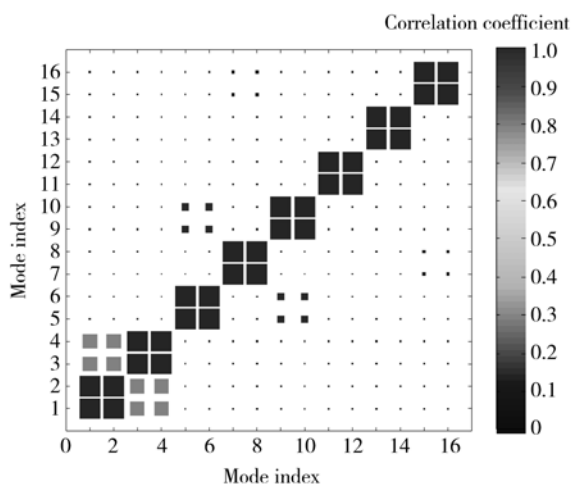


Fig. 11 Autocorrelation coefficients of the Zernike moment amplitude of the simplified aero-engine casing

The phase differences of the Zernike moments between the rotated mode shape and that of its reference account for the rotation angle of double mode shapes. For instance, mode 1 has same pattern as mode 2, just as in Fig. 12 shown. However, in Fig. 13, the phase angles of Zernike moment $Z_{2,2}$, which is the most largest one of the shape feature set, are different; and the other two significant components, $Z_{6,2}$ and $Z_{8,2}$, come into same truth. According to Eq. (12), by $Z_{4,2}$, the rotational angle is $\alpha = \frac{\arg(Z_{4,2}/Z'_{4,2})}{2} = -45^\circ$, that means mode 2

clockwise rotate 45° with respect to mode 1. An angle of -45° is also obtained by comparing the phase of the two Zernike moments $Z_{6,2}$ and $Z_{8,2}$. The rotated angles of other double mode pairs are list in Table 2.

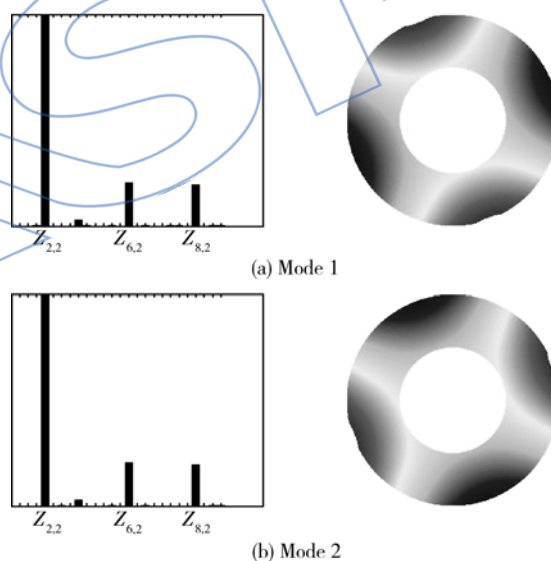
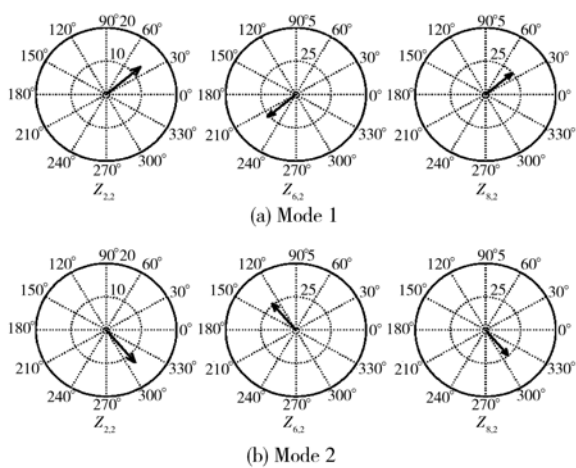


Fig. 12 Mode shape patterns and ZMDs of mode 1 and mode 2

Table 2 Rotated angles of double mode pairs

Double mode pair	Rotated angle/($^\circ$)
mode 3 vs mode 4	-45
mode 5 vs mode 6	30
mode 7 vs mode 8	22
mode 9 vs mode 10	-30
mode 11 vs mode 12	18
mode 13 vs mode 14	15
mode 15 vs mode 16	22.5

Fig. 13 $Z_{2,2}$, $Z_{6,2}$ and $Z_{8,2}$ of mode 1 and mode 2

2.3 Noise filtering using ZMD

In practical applications, the modal testing normally is imprecise due to the random and systematic noise. The challenge to be faced is that how to eliminate or reduce the noise effect in the measured vibration modes. Consider that Zernike moment transformation is robust to noise; it is a reasonable attempt to describe noised mode shapes using ZMD and rebuild more precise mode shapes which noise is eliminated or reduced.

In the present case, original mode shapes are interfused normal random noise. γ is the representation of intensity of noise, for instance, $\gamma=0.5$ represent 50% noise is added to original mode shapes. In this case, γ is set as 1.

It is clearly that distortion is appeared when noise is added into mode shapes, as Fig. 14 shown.

The precise mode shapes are reconstructed on the base of noise robustness property of Zernike moment. The noised mode shape patterns and filtered ones of three directions are shown in Fig. 15 to Fig. 17. $N_{\max}=8$ is adequate for our requirement during reconstruction.

Most mode shapes have restored original shapes after filtering noise, but distortion still present in some low order mode, as mode 1 to mode 4. From mode shape patterns filtered noise (Fig. 18), it is clear that the redundant moments affect the reconstruction of low order mode;

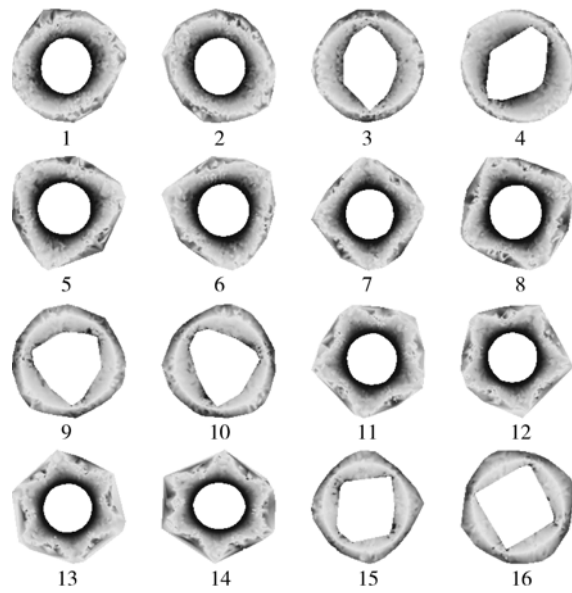


Fig. 14 Mode shape patterns contained noise of the aero-engine casing structure

however, those presenting in high order mode is inessential to whole mode shape feature set and do not work on the reconstruction.

The change of MAC values of noised and filtered mode shapes (all with original mode shapes) in three directions are also calculated, as Table 3 to Table 5 list.

Table 3 Changes of MAC of noised and filtered mode shapes in x direction

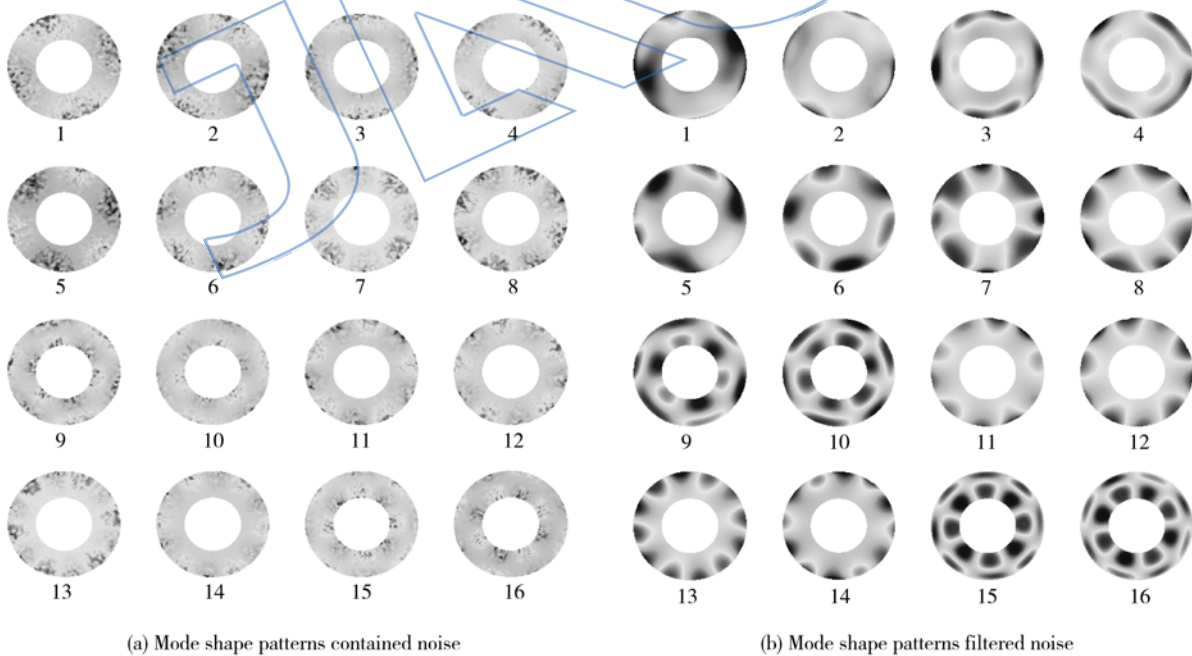
No. of mode	Noised	Filtered
1	0.51	0.84
2	0.53	0.82
3	0.48	0.74
4	0.53	0.73
5	0.50	0.85
6	0.51	0.88
7	0.45	0.89
8	0.60	0.94
9	0.50	0.89
10	0.49	0.90
11	0.50	0.92
12	0.51	0.96
13	0.46	0.95
14	0.47	0.95
15	0.54	0.87
16	0.48	0.85

Table 4 Changes of MAC of noised and filtered mode shapes in y direction

No. of mode	Noised	Filtered
1	0.50	0.77
2	0.44	0.73
3	0.50	0.77
4	0.50	0.77
5	0.50	0.82
6	0.52	0.86
7	0.54	0.85
8	0.50	0.94
9	0.55	0.76
10	0.45	0.75
11	0.53	0.89
12	0.60	0.90
13	0.46	0.91
14	0.47	0.94
15	0.54	0.79
16	0.48	0.77

Table 5 Changes of MAC of noised and filtered mode shapes in z direction

No. of mode	Noised	Filtered
1	0.48	0.68
2	0.50	0.76
3	0.52	0.76
4	0.48	0.78
5	0.44	0.77
6	0.51	0.80
7	0.52	0.90
8	0.44	0.90
9	0.53	0.76
10	0.50	0.75
11	0.48	0.93
12	0.53	0.93
13	0.50	0.90
14	0.50	0.93
15	0.51	0.78
16	0.48	0.78

**Fig. 15** Mode shape patterns contained noise and filtered ones in x direction

The MAC between noised and original mode shapes is nearly 0.5, the two have small correlation. After filtering, the MAC values are most raised up to 0.8, the more precise mode shapes

are obtained.

This case demonstrates that ZMD is a useful tool for eliminating or reducing the imprecision caused by noise.

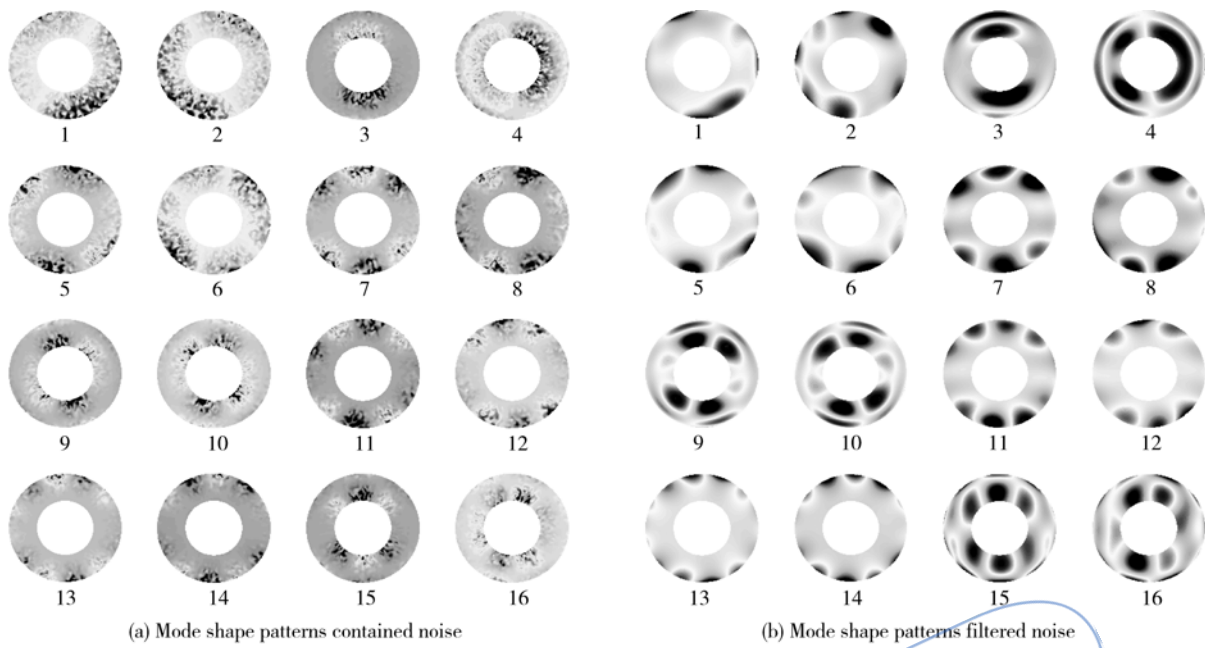


Fig. 16 Mode shape patterns contained noise and filtered ones in y direction

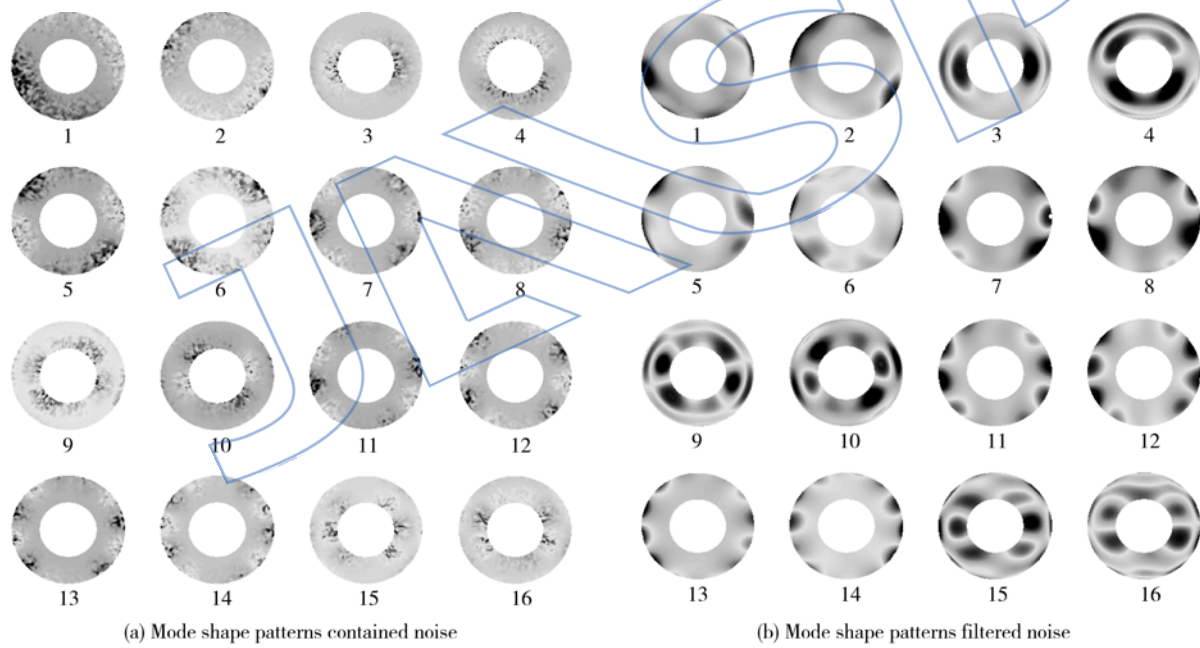


Fig. 17 Mode shape patterns contained noise and filtered ones in z direction

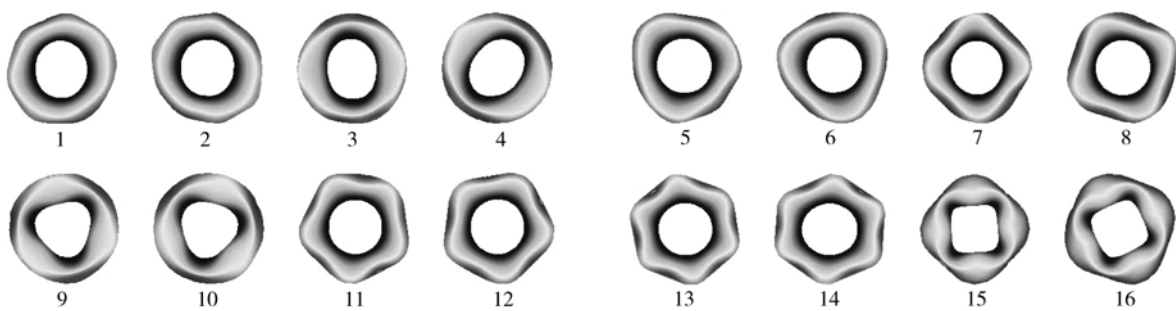


Fig. 18 Mode shape patterns filtered noise of the aero-engine casing structure

3 Conclusions

The mode shapes of the structure can be decomposed as a linear combination of a series of Zernike moment, with the feature of each Zernike moment reflecting the detail characteristic of mode shapes. Compared to traditional MAC method, the correlation method based on Zernike moment is more suitable for comparison of shapes of symmetric structure. Meanwhile, The noise robust property of the ZMDs makes it a powerful tool for removing or reducing the influence of noise in the mode shapes.

Simulation of the aero-engine casing structure indicates the advantage of this method to depict the mode shapes of a symmetric structure. Results demonstrate that the Zernike moment description of the mode shapes can not only effectively describe the double modes in the symmetric structure but also has the ability to obtain the angle between double mode pairs. Such features will be great helpful for model correlation and updating in the validation process of symmetric structure.

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